Multi-Resolution Variational Method for Ocean Current Estimation from SST and Altimetry Observations

Silèye BA(+), Ronan Fablet(+)
Emmanuelle Autret*, Bertrand Chapron*
Dpt Signaux et Communications(+), Telecom Bretagne, Brest
IFREMER*, Brest

08/03/2012
Geophysical variables

- **Sea surface temperature (SST)**
  - REMSS: low spatial resolution, low missing data rate
  - AVHRR-METOP: high spatial resolution, high missing data rate

- **Chlorophyll concentration (CHL)**
  - MERIS: spatial resolution, high missing data rate

- **Altimetry**
  - Low spatio-temporal resolution (1 observation/week)
Problem: missing data interpolation

- **Formal problem:**
  \[ Y_t = P_t X_t + \eta_t \]
  - Hidden state (complete geophysical map)
  - Observation with missing data
  - Projection operator
  - Gaussian noise \( \sim N(0, R) \)

- **Objective:**
  - recover the full geophysical maps
    - SST and altimetry

- **Our solution:**
  - Multi-resolution multi-modal variational data assimilation
Variational estimation: general principle

Data:
- Observation sequence \( Y_t, t \in [t_0, t_f] \) of geophysical variable \( X_t \)
- Dynamical model \( M(X_t) \): temporal evolution of \( X_t \)

Variational cost function
\[
J(X) = C(X,Y) + C(X)
\]

- \( C(X,Y) \): observation model
  - similarity between reconstructed variables and observations

- \( C(X) \): temporal consistency term
  - Consistency of variables temporal evolution
  - Build on top of the dynamical model \( M(X_t) \)
Variational model

State model:

\[ X = (\theta, \omega_g, u_a) \]

\[ \theta \quad \text{SST} \]

\[ \omega_g \quad \text{geostrophic vorticity} \]

\[ u_a \quad \text{ageostrophic velocity} \]

Dynamic model:

\[
\begin{aligned}
\partial_t \theta + (u_a + u(\omega_g)) \nabla \theta + \kappa \text{div}(\nabla \theta) \\
\partial_t \omega_g + u(\omega_g) \nabla \omega_g = \zeta_t \\
X_{t_0} = X_0 + v_t^0
\end{aligned}
\]

Variational Problem:

\[ J(\theta, \omega_g, u_a) = E_{\text{Obs}}(\theta, \omega_g) + E_{\text{Dynamic}}(\theta, \omega_g, u_a) + E_{\text{Dynamic}}(\omega_g) + E_{\text{MultiResol}}(\theta_{HR}, \theta_{LR}) \]

- observation term
- temporal consistency
Results: circulation estimation

- Convergence structures are better resolved with the geostrophic and the ageostrophic velocities

Black arrow = $U_a + U_g$
Magenta = $U_g$

Lyapunov exponents

- $U_g$
- $U_g + U_a$
Thanks for your attention
Questions & Commentaires
Single (SST) resolution variational method

- Hidden state sequence:
- Temporal evolution driven by the system

\[
\begin{aligned}
\partial_t \theta + M(\theta, \omega_g, u_a) &= \xi_t, \quad M(\theta, \omega_g, u_a) = (u_a + u(\omega_g))\nabla \theta + \kappa \text{div}(\nabla \theta) \\
\partial_t \omega_g + u(\omega_g) \nabla \omega_g &= \zeta_t \\
X_{t_0} &= X_0 + \nu_t
\end{aligned}
\]

- Variational problem

\[
J(\theta, \omega_g, u_a) = \int_{t_0}^{t_f} E(\theta, \omega_g) dt + \int_{t_0}^{t_f} \left\| \partial_t \theta - M(\theta, \omega_g, u_a) \right\|_{Q_\theta}^2 dt + \int_{t_0}^{t_f} \left\| \partial_t \omega_g - u(\omega_g) \nabla \omega_g \right\|_{Q_\omega}^2
\]

observation term

\[
E(\theta, \omega_g) = \sigma_\theta^{-1} \left\| Y^\theta - P_\theta \theta \right\|_2^2 + \beta \left\| \nabla \theta \right\|_2^2 + \sigma_\omega^{-1} \left\| Y^\omega - P_\omega \omega_g \right\|_2^2
\]

SST observation

vorticity observation (altimetry)
Multi-resolution variational method

- **Similar underlying dynamics**
  - Various geophysical variables follow the same ocean circulation
    - Same main frontal structures
  - Examples:
    - SST observation at various resolution (REMSS and METOP SST)

- **Solution:**
  - Multi-resolution variational processing with frontal structure constraints

- **Multi-resolution observation model:**

\[
E_{\theta}(\theta_1, \theta_2) = E(\theta_1) + E(\theta_2) + \int_{\Omega} g_{a}(|\nabla \theta_1|) \rho_\varepsilon \left( \frac{\nabla \theta_2 \cdot \nabla \theta_1^\perp}{|\nabla \theta_2| \cdot |\nabla \theta_1|} \right) dp
\]

\[
E(\theta) = \sigma^{-1} \left\| Y - P_{\theta} \right\|_2^2 + \beta \left\| \nabla \theta \right\|_2^2
\]

Independent cost

\[
g_{a}(x) = 1 - \frac{1}{1 + ax}
\]

Frontal structure marker (edge detector)

\[
\rho_\varepsilon(x) = \sqrt{x^2 + \varepsilon}
\]

Absolute value differentiable approximation

Regularity term: impose reconstructed variables to share main frontal structures
Results: frontal structure prior effects